The Coverage Problem in Three-Dimensional Wireless Sensor Networks

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Abstract—One of the fundamental issues in sensor networks is the *coverage* problem, which reflects how well a sensor network is monitored or tracked by sensors. In this paper, we formulate this problem as a decision problem, whose goal is to determine whether every point in the service area of the sensor network is covered by at least α sensors, where α is a given parameter and the sensing regions of sensors are modeled by balls (not necessarily of the same radius). This problem in a 2D space is solved in [1] with an efficient polynomial-time algorithm (in terms of the number of sensors). In this paper, we show that tackling this problem in a 3D space is still feasible within polynomial time. The proposed solution can be easily translated into an efficient polynomial-time distributed protocol.

I. INTRODUCTION

The rapid progress of wireless communication and embedded micro-sensing MEMS technologies has made *wireless sensor networks* possible. Such environments may have many inexpensive wireless nodes, each capable of collecting, storing, and processing environmental information, and communicating with neighboring nodes. In the past, sensors are connected by wire lines. Today, this environment is combined with the novel *ad hoc* networking technology to facilitate inter-sensor communication [2], [3]. The flexibility of installing and configuring a sensor network is thus greatly improved. Recently, a lot of research activities have been dedicated to sensor networks, including design issues related to the physical and media access layers [4], [5], [6] and routing and transport protocols [7], [8], [9]. Localization and positioning applications of wireless sensor networks are discussed in [10], [11], [12].

Since sensors may be spread in an arbitrary manner, two fundamental issues in a wireless sensor network are the *coverage problem* and the *deployment problem*. Given a sensor network, the coverage problem is to determine how well the sensing field is monitored or tracked by sensors, while the deployment problem is to address how to place sensors into a sensing field to meet certain coverage requirements. In the literature, these two problems have been formulated in various ways. In computational geometry, a *circle covering* [13] is an arrangement of circles on a plane that can fully cover the plane. The goal is to minimize the radius of circles, given a fixed number of circles. This issue is discussed in [14], [15] for square planes. Another related computational geometry issue is the *Art Gallery Problem* [16] which is to determine the number of observers such that every point

in the art gallery is monitored by at least one observer. Reference [17] proposes polynomial time algorithms to find the *maximal breach path* and the *maximal support path* that are least and best monitored in the sensor network. How to find the *minimal and maximal exposure path* that takes the duration that an object is monitored by sensors is addressed in [18], [19]. On the other hand, some works are targeted at particular applications, but the central idea is still related to the coverage issue. For example, to reduce sensors' on-duty time, those sensors that share the common sensing region and task may be turned off to conserve energy and thus to extend the network lifetime [20], [21], [22]. Placement of static sensors is discussed in [23], [24], while how to achieve better deployment of mobile sensors is addressed by [25], [26].

In this paper, we consider a general geometric problem related to the coverage and deployment issues: Given a set of sensors in a 3D sensing field, we want to determine if this field is sufficiently α -covered, where α is a given integer, in the sense that every point in the field is covered by at least α sensors. The sensing range of each sensor is modeled by a 3D ball. While most applications may require $\alpha = 1$, applications requiring $\alpha > 1$ may occur in situations where a stronger environmental-monitoring or fault-tolerating capability is desired, such as military applications. Some applications may require multiple sensors to detect an event. For example, the triangulation-based positioning protocols [11], [12] require at least three sensors (i.e., $\alpha \geq 3$) at any moment to monitor a moving object.

The 2-dimensional coverage problem has been solved efficiently in [1] with a polynomial time algorithm, in terms of the number of sensors. At the first glance, the 3-dimensional coverage problem seems very difficult since even determining the subspaces divided by the spheres of sensors' sensing ranges is very complicated. However, in this paper, we show that tackling this problem is still feasible within polynomial time. Reference [21] also considers the same coverage problem in a 2D space combined with the communication connectivity issue. However, it incurs higher computational complexity to determine the network's coverage level compared to [1]. The *arrangement* issue [27], [28], which is widely studied in combinatorial and computational geometry, also considers how a finite collection of geometric objects decomposes a space into connected elements. However, to construct arrangements, only *centralized* algorithms are proposed in the literature,

whilst what we need for a wireless sensor network is a distributed solution. The solution proposed in this paper can be easily translated to a distributed protocol where each sensor only needs to collect local information to make its decision.

We propose a novel solution by reducing the geometric problem from a 3D space to a 2D space, and further to a 1D space, thus leading to a very efficient solution. In essence, our solution tries to look at how the sphere of each sensor's sensing range is covered. As long as the spheres of all sensors are sufficiently covered, the whole sensing field is sufficiently covered. To determine whether each sensor's sphere is sufficiently covered, we in turn look at how each spherical cap and how each circle of the intersection of two spheres is covered. By stretching each circle on a 1-dimensional line, the level of coverage can be easily determined.

This paper is organized as follows. Section II gives some preliminaries. Our solution is presented in Section III. Section IV draws our conclusions.

II. PRELIMINARIES AND PROBLEM STATEMENT

We are given a set of sensors, $S = \{s_1, s_2, \dots, s_n\}$, in a three-dimensional cuboid sensing field A . Each sensor $s_i, i = 1 \dots n$, is located at coordinate (x_i, y_i, z_i) inside A and has a sensing range of r_i . So each sensor s_i 's sensing area is a ball centered at (x_i, y_i, z_i) with radius r_i , denoted as $B_i = (x_i, y_i, z_i, r_i)$. The sphere of B_i is the surface of B_i , denoted as S_i .

Consider two sensors s_i and s_j which have non-empty intersecting sensing regions. The *spherical cap* $Cap(i, j)$ is the intersection of sphere S_i and ball B_j . The *circle* $Cir(i, j)$ is the intersection of spheres S_i and S_j . The *center of spherical cap* $Cap(i, j)$, denoted by $Cen(i, j)$, is the intersection of line $\overleftrightarrow{s_i s_j}$ and spherical cap $Cap(i, j)$. Given any two points p and p' on S_i , the *geodesic distance between p and p'* , denoted by $GD(p, p')$, is the minimum great circle distance between p and p' on S_i . The *radius of $Cap(i, j)$* , denoted by $Rad(i, j)$, is $GD(Cen(i, j), p)$, where p is any point on $Cir(i, j)$. Examples of these terms are illustrated in Fig. 1.

Definition 1: A point in A is said to be *covered* by s_i if it is within s_i 's sensing range. A point in A is said to be α -covered if it is covered by at least α distinct sensors.

Definition 2: Given a natural number α , the α -Ball-Coverage (α -BC) Problem is a decision problem whose goal is to determine whether all points in A are α -covered or not.

III. THE PROPOSED SOLUTION

In the section, we propose an algorithm to solve the α -BC problem with time complexity $O(nd^2 \log d)$, where d is the maximum number of sensors whose sensing ranges may intersect a given sensor's sensing range. Our approach does not try to look at how each point (or subspace) in A is covered by sensors because determining how A is divided by n spheres is much too complicated. Instead, our algorithm tries to determine whether the sphere of a sensor under consideration is sufficiently covered. Further, to determine whether each sensor's sphere is sufficiently covered, we look at how the

Fig. 1. Illustration of terminologies.

circle of each spherical cap of a sensor intersected by its neighboring sensors is covered. By collecting this information from all sensors, a correct answer can be obtained. Intuitively, we reduce the decision problem from a 3D space to one in a 2D space, and then to one in a 1D space.

A. Theoretical Fundamentals

Observe that the sensing field A is divided into a number of subspaces by sensors' spheres. Each subspace's surface consists of a number of spherical segments. Because of the continuity nature, the level of coverage of a subspace can actually be derived from those of its spherical segments. Furthermore, each spherical segment must be bounded by a number of circle segments on some spherical caps. By the continuity nature again, the level of coverage of a spherical segment can actually be derived from those of its circle segments that bound the spherical segment. This is how we reduce the problem from a 3D space to a 2D space, and then to a 1D space. In the following discussion, we will use "subspace", "spherical segment", and "circle segment" to facilitate our presentation.

Definition 3: Consider any two sensors s_i and s_j . A point on sphere S_i is *sphere-covered* by s_j if it is on or within sphere S_j . We say that s_i is α -sphere-covered if all points on sphere S_i are sphere-covered by at least α other sensors.

Lemma 1: If a sphere S_i is α -sphere-covered, then each subspace that is adjacent to S_i is at least α -covered.

Proof: Since sphere S_i is α -sphere-covered, by definition each subspace that is adjacent to S_i but outside S_i is also α -covered. The subspaces inside S_i are at least $(\alpha + 1)$ -covered because they are further covered by s_i ¹. $\square$

Theorem 1: If each sphere is α -sphere-covered, then the sensing field A is α -covered.

¹In most cases, the subspaces inside S_i are $(\alpha + 1)$ -covered. However, in the special case that there are k other sensors colocating with s_i and having the same sensing radiuses with s_i , these subspaces will be $(\alpha + k + 1)$ -covered.

Fig. 2. The relationship between $Cap(i, j)$ and $Cap(i, k)$: case 1.

Proof: Observe that each subspace in A must be bounded by some spherical segments. Since each sphere is α -sphere-covered, by Lemma 1 all subspaces are at least α -covered, which proves this theorem. $\square$

Below, to facilitate our presentation, we translate sphere coverage into cap coverage. This allows us to look at a single sphere when examining coverage.

Definition 4: Consider any sensor s_i and its neighboring sensor s_j . A point p on S_i is *cap-covered* by $Cap(i, j)$ if p is on $Cap(i, j)$. Point p is α -cap-covered if it is cap-covered by at least α caps on S_i .

Corollary 1: Consider any sensor s_i . If each point on S_i is α -cap-covered, then sphere S_i is α -sphere-covered.

Proof: This corollary can be easily proved by observing the equivalence between the definitions of sphere coverage and cap coverage. $\square$

Definition 5: Consider any sensor s_i and two of its neighboring sensors s_j and s_k . We say that a point p on $Cir(i, j)$ is *circle-covered* by $Cap(i, k)$ if p is cap-covered by $Cap(i, k)$. We say that the spherical circle $Cir(i, j)$ is α -circle-covered if every point on $Cir(i, j)$ is circle-covered by at least α caps on S_i other than $Cap(i, j)$.

Lemma 2: Consider any sensor s_i and its neighboring sensor s_j . If circle $Cir(i, j)$ is α -circle-covered, then each spherical segment on S_i that is adjacent to $Cir(i, j)$ is at least α -cap-covered.

Proof: Since circle $Cir(i, j)$ is α -circle-covered, each spherical segment on S_i that is adjacent to $Cir(i, j)$ but outside $Cap(i, j)$ is also α -cap-covered. The spherical segments on S_i inside $Cap(i, j)$ are at least $(\alpha + 1)$ -cap-covered because they are further covered by $Cap(i, j)$ ². $\square$

Theorem 2: Consider any sensor s_i and each of its neighboring sensors s_j . If each circle $Cir(i, j)$ is α -circle-covered, then the sphere S_i is α -cap-covered.

²In most cases, these spherical segments are $(\alpha + 1)$ -cap-covered. However, in the special case that there are k other caps colocating with the current $Cap(i, j)$, these spherical segments will be $(\alpha + k + 1)$ -cap-covered. Note that colocating caps may appear when two spheres intersect with another sphere on the same circle.

Proof: Observe that each spherical segment on S_i must be bounded by some circle segments. Since each circle is α -cap-covered, by Lemma 2 all spherical segments on S_i are at least α -cap-covered, which proves this theorem. $\square$

B. Determining the Intersection of Spherical Caps

The above derivation implies that to determine how A is covered, it is sufficient to determine how each circle is covered. To determine circle coverage, consider any two spherical caps $Cap(i, j)$ and $Cap(i, k)$ on sphere S_i of a sensor s_i . There are two cases:

- 1: The center of $Cap(i, k)$, $Cen(i, k)$, is inside $Cap(i, j)$, i.e., $GD(Cen(i, j), Cen(i, k)) \leq Rad(i, j)$.
 - (i) If $Rad(i, k) < Rad(i, j) - GD(Cen(i, j), Cen(i, k))$, then $Cap(i, j)$ is not circle-covered by $Cap(i, k)$ (refer to Fig. 2(a)).
 - (ii) If $Rad(i, j) - GD(Cen(i, j), Cen(i, k)) \leq Rad(i, k) \leq GD(Cen(i, j), Cen(i, k)) + Rad(i, j)$, then the arch of $Cir(i, j)$ falling in the angle $[\lambda, \lambda + \theta]$ is circle-covered by $Cap(i, k)$ (refer to Fig. 2(b)).
 - (iii) If $Rad(i, k) > GD(Cen(i, j), Cen(i, k)) + Rad(i, j)$, then the whole range $[0, 2\pi]$ of $Cap(i, j)$ is circle-covered by $Cap(i, k)$ (refer to Fig. 2(c)).
- 2: The center of $Cap(i, k)$, $Cen(i, k)$, is outside $Cap(i, j)$, i.e., $GD(Cen(i, j), Cen(i, k)) > Rad(i, j)$.
 - (i) If $Rad(i, k) < GD(Cen(i, j), Cen(i, k)) - Rad(i, j)$, then $Cap(i, j)$ is not circle-covered by $Cap(i, k)$ (refer to Fig. 3(a)).
 - (ii) If $GD(Cen(i, j), Cen(i, k)) - Rad(i, j) \leq Rad(i, k) \leq GD(Cen(i, j), Cen(i, k)) + Rad(i, j)$, then the arch of $Cir(i, j)$ falling in the angle $[\lambda, \lambda + \theta]$ is circle-covered by $Cap(i, k)$ (refer to Fig. 3(b)). Note that it is possible that there is no intersection between $Cir(i, j)$ and $Cir(i, k)$, but $Cir(i, j)$ is fully covered by $Cap(i, k)$, as illustrated in Fig. 3(c).
 - (iii) If $Rad(i, k) > GD(Cen(i, j), Cen(i, k)) + Rad(i, j)$, then the whole range $[0, 2\pi]$ of $Cir(i, j)$ is circle-covered by $Cap(i, k)$ (refer to Fig. 3(d)).

Fig. 3. The relationship between $Cap(i, j)$ and $Cap(i, k)$: case 2.

C. The Complete Algorithm

Below, we propose an $O(d^2 \log d)$ algorithm to determine whether a sensor is α -sphere-covered or not. The algorithm can be executed either in a centralized or in a fully distributed manner independently by each sensor. First, each sensor has to collect how its neighboring sensors intersect with itself and calculate the corresponding spherical caps. Next, it has to figure out the relationship between spherical caps, as described above. Then we can determine the level of circle coverage of each circle. After each cap's circle coverage level is determined, the sensor's sphere coverage level can be found out, which in turn gives the overall coverage of A . The detail algorithm to be run by each sensor s_i is listed below.

- 1) For each neighboring sensor s_j of s_i , do the following.
 - a) Calculate the circle $Cir(i, j)$ of $Cap(i, j)$.
 - b) For each neighbor $s_k \neq s_j$ of s_i , we determine how $Cap(i, k)$ intersects with $Cir(i, j)$. Specifically, we calculate the angle of $Cir(i, j)$ that is circle-covered by $Cap(i, k)$, denoted by $[\theta_{k,L}^j, \theta_{k,R}^j]$.
 - c) For all angles $[\theta_{k,L}^j, \theta_{k,R}^j]$ found in step b), place points $\theta_{k,L}^j$ and $\theta_{k,R}^j$ on a line segment $[0, 2\pi]$, and then sort all these points in an ascending order into a list L_j .

- d) (sketched) Traverse the line segment $[0, 2\pi]$ by sequentially visiting each point in the sorted list L_j to determine the circle coverage of $Cir(i, j)$, denoted by cc_j .

end for.

- 2) The sphere coverage of s_i is the minimum circle coverage of all circles on S_i , i.e., $\min_{\text{neighbor } s_j} \{cc_j\}$.

Let d be the maximum number of sensors neighboring to a sensor ($d \leq n$). Step 1a, 1b, 1c, and 1d have time complexities of $O(1)$, $O(d)$, $O(d \log d)$, and $O(d)$, respectively. So the complexity of step 1 is $O(d^2 \log d)$, which is also the complexity of the whole algorithm for one sensor.

The step 1d, though sketched, can be easily implemented as follows. Whenever an element $\theta_{k,L}^j$ is traversed, the level of coverage should be increased by one. Whenever an element $\theta_{k,R}^j$ is traversed, the level of coverage should be decreased by one. An example is shown in Fig. 4. The point on angle 0 can be easily determined to be 3. When visiting points c, d, f, h, j, l, n , and p (resp., points a, b, e, g, i, k, m , and o), the level of coverage should be increased (resp., decreased) by 1.

Below, we comment on several special cases, which we leave not addressed on purpose for simplicity in the above

Fig. 4. An example to determine the coverage of a circle.

discussion. First, it is possible that a sensor's sensing range is fully covered by another sensor's, i.e., a sensing ball is entirely inside another sensing ball. These two spheres do not have any intersection. Alternatively, we can regard the whole sphere of the smaller one as a special spherical cap. So we can simply increase the sphere coverage level of the smaller sphere by one after executing our algorithm. Another boundary case is that some sensors' sensing ranges may exceed the sensing field A . If so, we can simply assign the spherical segments falling outside A as ∞ -sphere-covered.

IV. CONCLUSIONS

We have proposed a solution to the three-dimensional coverage problem for wireless sensor networks. We have shown that tackling this problem in a 3D space can be done at polynomial time. The result may be used in deploying sensors in 3D space and in reducing on-duty time of wireless sensors.

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